A new two-potential formalism for the Maxwell equations and its application to numerical simulation of electromagnetic processes

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A new formulation of electromagnetic field equations based on the use of two vector potentials and two scalar potentials is proposed. This formulation allows the Maxwell equations both in vacuum and in a material medium to be written in the form of a hyperbolic system possessing a number of desirable properties. In particular, in contrast to the original Maxwell equations, the new system of equations consists only of evolutionary equations and does not include relations having the character of differential constraints. Such differential constraints (divergence-free conditions for electric and magnetic field) lead to significant difficulties when the Maxwell equations in the standard formulation are solved numerically with finite-difference methods. All eigenvalues of the Jacobi matrix of the derived system of equations corresponds to physical modes propagating with the velocity of light; there are no non-physical modes corresponding to the zero eigenvalue and obtained in the frequently used approach where only equations containing vector field rotors are solved, whereas equations with divergence are ignored. The relativistic invariance of the new formalism is shown.

All these facts allow powerful advanced shock-capturing methods based on approximation of spatial derivatives by upwind differences to be used to solve the new system numerically. Examples of numerical simulations of propagation of electromagnetic waves by solving the equations in the new formulation are given. One of the high-resolution numerical methods, a fifth-order WENO (Weighted Essentially Non-Oscillatory scheme, is employed and the results obtained are compared with the standard second-order FDTD (finite-difference time-domain) method. It is demonstrated that such a numerical approach allows the solution to be obtained with high accuracy, including problems that involve jumps of material properties of the medium.

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